Arithmetic Sequences (Explicit Formula)

For the following patterns, find the next two numbers. Then describe the rule you are applying each time.

Pattern	Rule	Common Difference
a. -4, -2, 0, 2, 4, 6, ... (+2 +2 +2)	adding 2	+2
b. 6.5, 5, 3.5, 2, 0.5, -1, ...	subtracting 1.5	-1.5
c. 12, 18, 24, 30, 36, ...	adding 6	+6
d. 11, 9, 7, 5, 3, ...	subtracting 2	-2

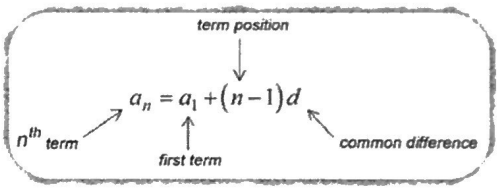
e. What did you notice about your patterns? same # being +/- each time

f. What do you think the "..." means? continues on forever

A **Sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A sequence in which you get the next consecutive term by adding or subtracting a constant value is called an **arithmetic sequence**. In other words, we just add or subtract the same value over and over...infinitely. This constant value is called the **common difference**

Formula for Arithmetic Sequences

Explicit Formula:



Why We Have a Formula for Sequences

Take a look at the following pattern: 1 2 3 4 5
4, 8, 12, 16, ... 20
+4 +4 +4

What is the common difference? +4

What is the 1st term? 4 What is the 3rd term? 12 What is the 5th term? 20

What is the 54th term? / (You don't want to add 4 over and over 54 times?!?!?!?)

This is why the **Explicit Formula** was created – as long as you know your common difference and 1st term, you can create a rule to describe any arithmetic sequence and use it to find any term you want.

Creating an Explicit Rule	
1. Write down the Explicit Formula.	1. $a_n = a_1 + (n-1)d$
2. Substitute the first term in for a_1 and common difference in for d.	2. $a_n = 4 + (n-1)4$
3. Simplify the right side of the equation by distributing and combining like terms. Your result should be an equation that looks very similar to y = mx + b (except it will look more like $a_n = dn + c$).	3. $a_n = 4 + 4n - 4$ $a_n = 4n + 0$ $a_n = 4n$
4. To find an nth term, substitute the term number you are wishing to find into n. Find a_{54}.	4. $a_{54} = 4(54) = 216$

Creating an Explicit Rule

Write an Explicit Rule for the following sequences:

a. 1, 8, 15,... (+7 +7)

$a_1 =$ 1

$d =$ +7

1.) $a_n = a_1 + (n-1)d$

2.) $a_n = 1 + (n-1)7$

3.) $a_n = 1 + 7n - 7$

$a_n = 7n - 6$

b. 4, 0, -4,... (-4 -4)

$a_1 =$ 4

$d =$ -4

1.) $a_n = a_1 + (n-1)d$

2.) $a_n = 4 + (n-1)-4$

3.) $a_n = 4 + -4n + 4$

$a_n = -4n + 8$

Finding the Nth Term

To find the nth term, particularly when the nth term is quite large, you want to create an Explicit Rule first and then substitute that term number into the rule for n.

~~For the given sequences, create an explicit rule and then~~ use the rule (from above) to find the following terms:

a. ~~5, 10, 15, 20,~~ ..... Find 21st term or a_{21}

$a_n = 7n - 6$

$a_{21} = 7(21) - 6$

$a_{21} = 147 - 6$

$a_{21} = 141$

b. ~~30, 22, 14, 6,~~ .... Find 30th term or a_{30}

$a_n = -4n + 8$

$a_{30} = -4(30) + 8$

$a_{30} = -120 + 8$

$a_{30} = -112$

Finding Terms Using an Explicit Rule

For the following sequences, find the first ~~three~~ two terms: a_1, a_2

a. $a_n = 4 + 3(n-1)$

$a_1 = 4 + 3(1-1)$

$a_1 = 4 + 3(0)$

$a_1 = 4 + 0 = 4$

$a_2 = 4 + 3(2-1)$

$a_2 = 4 + 3(1)$

$a_2 = 4 + 3 = 7$

b. $a_n = -(n-1)$

$a_1 = -1(1-1)$

$a_1 = -1(0) = 0$

$a_2 = -1(2-1)$

$a_2 = -1(1) = -1$

Arithmetic Sequences (Recursive Formula)

There is a second formula for arithmetic sequences called the **Recursive Formula.** The recursive formula allows you to find the next term in a sequence if you know the common difference and any term of the sequence

$$a_n = a_{n-1} + d$$

Nth Term **Previous Term** **Common Difference**

Finding Terms Using a Recursive Formula

For the following recursive formulas, find the first three terms: a_1, a_2, a_3

a. $a_1 = 4$ ✓
$a_n = a_{n-1} + 4$

$a_2 = a_{2-1} + 4$
$a_2 = a_1 + 4$
$a_2 = 4 + 4 = 8$

$a_3 = a_{3-1} + 4$
$a_3 = a_2 + 4$
$a_3 = 8 + 4 = 12$

4, 8, 12
$d = +4$

b. $a_1 = -7$ ✓
$a_n = a_{n-1} - 6$

$a_2 = a_{2-1} - 6$
$a_2 = a_1 - 6$
$a_2 = -7 - 6 = -13$

$a_3 = a_{3-1} - 6$
$a_3 = a_2 - 6$
$a_3 = -13 - 6$
$a_3 = -19$

$-7, -13, -19$
$d = -6$

c. $a_1 = -3.5$ ✓
$a_n = a_{n-1} + 9$

$a_2 = a_{2-1} + 9$
$a_2 = a_1 + 9$
$a_2 = -3.5 + 9 = 5.5$

$a_3 = a_{3-1} + 9$
$a_3 = a_2 + 9$
$a_3 = 5.5 + 9 = 14.5$

$-3.5, 5.5, 14.5$
$d = +9$

Creating a Recursive Rule

For the following sequences, create a recursive rule:

a. 1, 8, 15,... (+7 +7)
$d = +7$
$a_n = a_{n-1} + 7$

b. 4, 0, -4,... (-4 -4)
$d = -4$
$a_n = a_{n-1} - 4$

c. -5, 3, 11,... (+8 +8)
$d = +8$
$a_n = a_{n-1} + 8$