Unit Rates

A **unit rate** is a comparison of two quantities in which the denominator has a value of one unit. To calculate a unit rate, just divide the numerator by the denominator. Unit rates are helpful in real life for determining the best buy, most miles per gallon, the fastest car, cellphone, etc, and many other uses. Take a look at the following example:

A car dealership advertised the following rates on gal mileage for three new cars:

The Avalon can travel 480 miles on 10 gallons of gas. $\frac{480 \text{ miles} \div 10}{10 \text{ gallons} \div 10} = \frac{48 \text{ miles}}{1 \text{ gallon}}$

The Compass can travel 400 miles on 8 gallons of gas.

The Patriot can travel 360 miles on 9 gallons of gas.

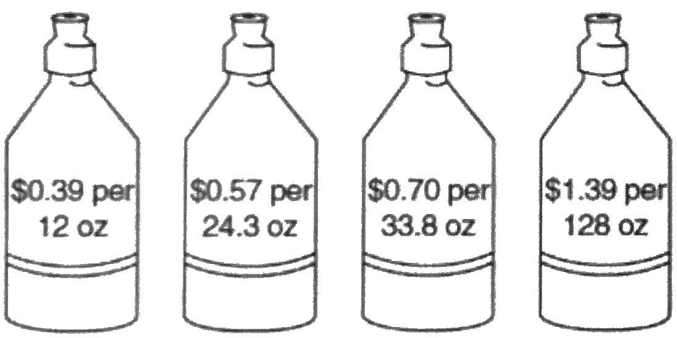
Which car gets the best gas mileage? Change each ratio to a unit rate to help make your decision.

Avalon: $\frac{480 \text{ miles} \div 10}{10 \text{ gallons} \div 10} = \frac{48 \text{ miles}}{1 \text{ gallon}}$ ★★ Compass: $\frac{400 \text{ miles}}{8 \text{ gallon}} = \frac{50 \text{ miles}}{1 \text{ gallon}}$ Patriot: $\frac{360 \text{ miles}}{9 \text{ gallon}} = \frac{40 \text{ miles}}{1 \text{ gallon}}$

Practice: Using unit rates, determine the best buy.

a. Round to thousandths place!

Bottle 1 Bottle 2 Bottle 3 Bottle 4 ★★

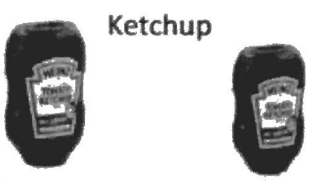


④ $\frac{\$1.39}{128 \text{ oz}} = \frac{\$0.011}{1 \text{ oz}}$

① $\frac{\$0.39}{12 \text{ oz}} = \frac{\$0.0325}{1 \text{ oz}}$ ② $\frac{\$0.57}{24.3 \text{ oz}} = \frac{\$0.0235}{1 \text{ oz}}$ ③ $\frac{\$0.70}{33.8 \text{ oz}} = \frac{\$0.0207}{1 \text{ oz}}$

b.

Ketchup



★★ 32 oz - $3.29 24 oz - $2.59

$\frac{\$3.29}{32 \text{ oz}} = \frac{\$0.103}{1 \text{ oz}}$ $\frac{\$2.59}{24 \text{ oz}} = \frac{\$0.108}{1 \text{ oz}}$

Unit rates are also helpful for calculating multiple numbers of an item (like when you are at the grocery store)

a. If a pound of bananas costs $0.53 a pound, how much are 4 pounds of bananas?

$\frac{\$0.53}{1\text{ lb}} \overset{\times 4}{=} \frac{x}{4\text{ lb}}$ $\quad x = \$2.12$

b. If a box of Cheerios costs $2.99, how much are 3 boxes of Cheerios?

$\frac{\$2.99}{1\text{ box}} \overset{\times 3}{=} \frac{x}{3\text{ boxes}}$ $\quad x = \$8.97$

c. If milk costs $2.59 a gallon, how much will 7 gallons cost?

$\frac{\$2.59}{1\text{ gallon}} \overset{\times 7}{=} \frac{x}{7\text{ gallons}}$ $\quad x = \$18.13$

Problem Solving with Unit Rates

a. Anne is painting her house light blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue?

$\frac{3\text{ cans white}}{2\text{ cans blue}} \overset{\times 3}{=} \frac{x}{6\text{ cans blue}}$ $\quad x = 9\text{ cans white}$

b. Ryan is making a fruit drink. The directions say to mix 5 cups of water with 2 scoops of powdered fruit mix. How many cups of water should he use with 9 scoops of fruit mix?

$\frac{5\text{ cups}}{2\text{ scoops}} \not= \frac{x}{9\text{ scoops}}$

$9 \cdot 5 = 2 \cdot x$

$45 = 2x$

$22.5\text{ cups} = x$ → $22\frac{1}{2}$ cups water

c. A publishing company is looking for new employees who can type at least 45 words per minute. Jessie can type 704 words in 16 minutes. Does she type fast enough to qualify for the job?

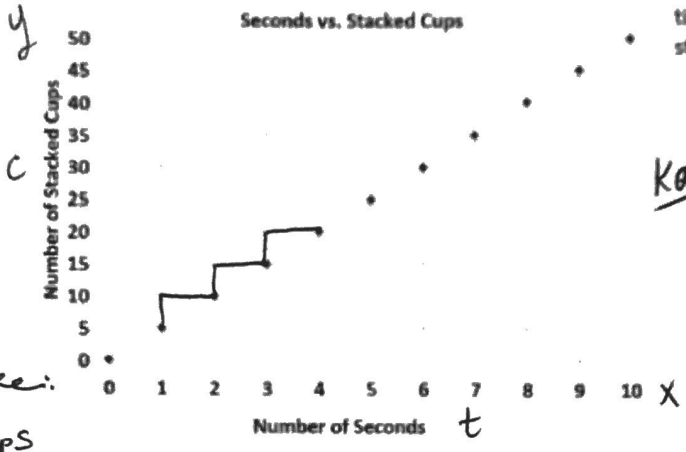
$\frac{45\text{ words}}{1\text{ minute}}$

$\frac{704\text{ words}}{16\text{ minutes}} = \frac{x}{1\text{ minute}}$ $\quad x = 44\text{ words} - \text{NO!}$

Using Unit Rates on a Graph

Claire & Kate entered a cup stacking contest so they have been practicing. Below is a graph of their progress.

Claire:



Kate: $c = 4t$, where t represents the amount of time in seconds and c represents the number of stacked cups.

cups / seconds

a. At what rate does each girl stack her cups during the practice session?

Kate: $c = 4t$ → $\frac{4 \text{ cups}}{1 \text{ sec}}$ Claire: $c = 5t$ → $\frac{5 \text{ cups}}{1 \text{ sec}}$

b. Kate notices she is not stacking her cups fast enough. What would Kate's equation look like if she wanted to stack cups faster than Claire?

$c = 6t$

Claire: $\frac{5 \text{ cups}}{1 \text{ sec}}$ $y = 5x$ OR $c = 5t$

Emilio was to buy a new motorcycle. He wants to base his decision off the gas efficiency for each motorcycle. Which motorcycle is more gas efficient?

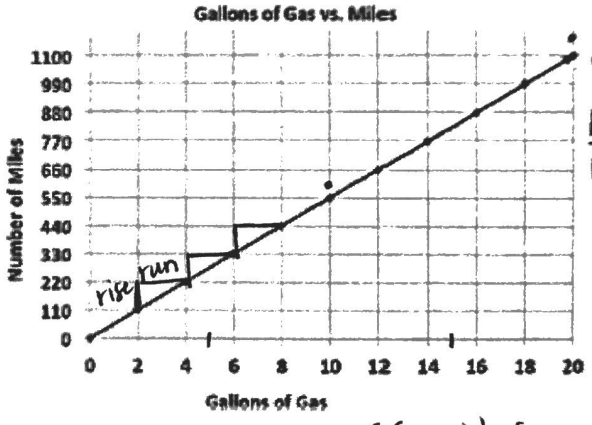
Sports Motorcycle:

Number of Gallons of Gas	5	10	15	20
Number of Miles	287.5	575	862.5	1,150

Leisure:

# gallons	2	4	6	8
# miles	110	220	330	440

Leisure Motorcycle:



Slope = $\frac{\text{Rise}}{\text{Run}} = \frac{110 \text{ m}}{2 \text{ g}} = \frac{55 \text{ m}}{1 \text{ g}}$

1 Unit Rate

Sports: $\frac{287.5 \text{ miles}}{5 \text{ gallons}} = \frac{57.5 \text{ miles}}{1 \text{ gallon}}$

Leisure: $\frac{110 \text{ miles}}{2 \text{ gallons}} = \frac{55 \text{ miles}}{1 \text{ gallon}}$

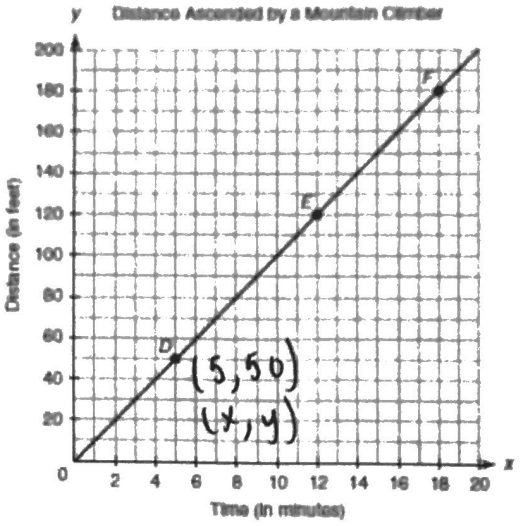
When viewing a unit rate on a graph, you are essentially looking at the slope of the line!!

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

$\frac{y}{x}$

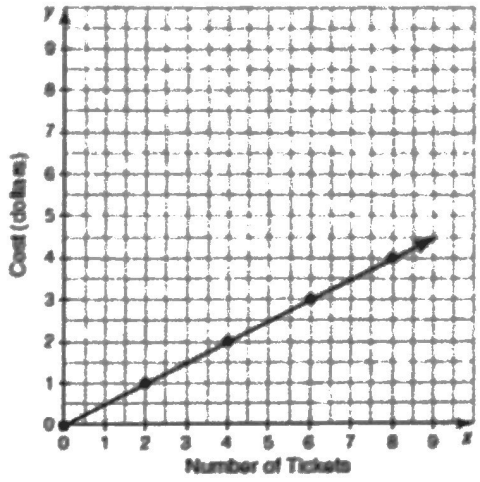
Practice: Calculate the slope (unit rate) of each graph:

a.



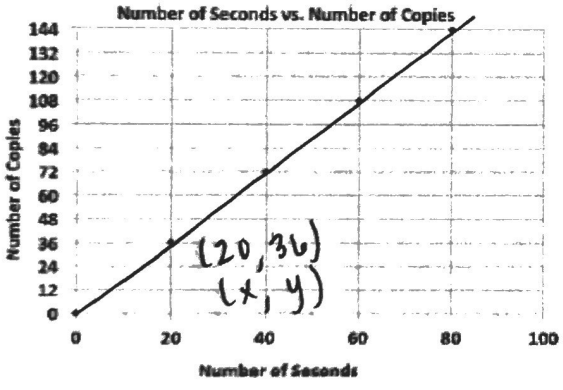
$\frac{y}{x} = \frac{50 \text{ feet}}{5 \text{ min}} = \frac{10 \text{ Feet}}{1 \text{ min}}$

b.



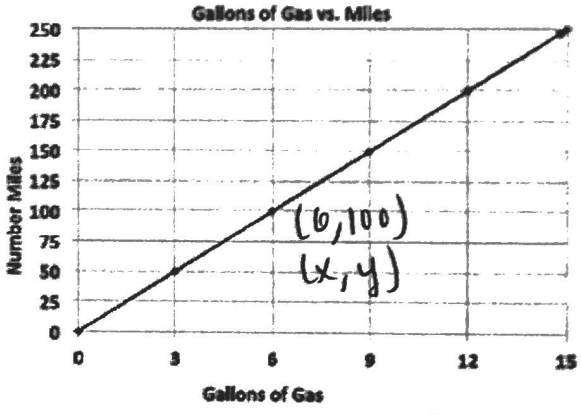
$\frac{y}{x} = \frac{\$1.00}{2 \text{ tickets}} = \frac{\$0.50}{1 \text{ ticket}}$

c.



$\frac{y}{x} = \frac{36 \text{ seconds}}{20 \text{ copies}} = \frac{1.8 \text{ seconds}}{1 \text{ copy}}$

d.



$\frac{y}{x} = \frac{100 \text{ miles}}{6 \text{ gallons}} = \frac{\approx 16.67 \text{ miles}}{1 \text{ gallon}}$