

## Functions

A **relation** is a relationship between two or more data sets that can be compared

A **function** is a patterned relation that maps each input to one and only one output, which means a function has no input with more than one output (No x-values going to two different y-values). The first coordinate of an ordered pair (x) in a relation is the **input**, and the second coordinate (y) is the **output**. We refer to the set of all inputs as the **domain** and the set of all outputs as the **range**.

Determine if the following are functions. Then state the domain and range:

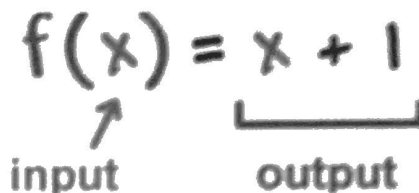
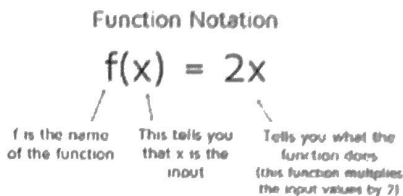
<p>a.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: left;"> <p>Input</p> <p>(1, 7) (2, 1) (3, 5) (4, 3)</p> </div> <div style="text-align: left;"> <p>Output</p> </div> </div>	<p>b. <math>\{(3, 4), (9, 8), (3, 7), (4, 20)\}</math></p> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span><math>\bar{x}</math></span> <span><math>\bar{x}</math></span> <span><math>\bar{x}</math></span> <span><math>\bar{x}</math></span> </div>	<p>c.</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="width: 50px;">Input</th> <th style="width: 50px;">Output</th> </tr> </thead> <tbody> <tr><td style="text-align: center;">-10</td><td style="text-align: center;">20</td></tr> <tr><td style="text-align: center;">-5</td><td style="text-align: center;">10</td></tr> <tr><td style="text-align: center;">0</td><td style="text-align: center;">0</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">10</td></tr> <tr><td style="text-align: center;">10</td><td style="text-align: center;">20</td></tr> </tbody> </table>	Input	Output	-10	20	-5	10	0	0	5	10	10	20
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Function or Not a Function	Function or Not a Function	Function or Not a Function												
Domain: $\{1, 2, 3, 4\}$	Domain: $3, 9, 4$	Domain: $-10, -5, 0, 5, 10$												
Range: $\{7, 1, 5, 3\}$	Range: $4, 8, 7, 20$	Range: $20, 10, 0, 10, 20$												
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<p>h. d. (social security number, person)</p>	<p>i. e. (shoe size, person)</p>	<p>j. f. (zip code, person)</p>												
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### Different Meanings of Domain and Range Organizer

<b>D</b>	Domain	<b>R</b>	Range
<b>I</b>	Inputs	<b>O</b>	Outputs
<b>X</b>	X-Values	<b>Y</b>	Y-Values
<b>I</b>	Independent Variable	<b>D</b>	Dependent Variable

### Function Notation

If  $x$  is the independent variable and  $y$  is the dependent variable, then function notation for  $y$  is  $f(x)$ , which is read "f of x," where  $f$  names the function. When an equation is in two variables and  $f$  describes a function, you can use function notation to write it:



The following problems are written in **function notation**.

$$f(x) = 3x + 1$$

$$f(x) = x^2 + 3x - 1$$

$$f(x) = 2x^2 + x - 1$$

$$f(a) = 3a + 1$$

$$f(\heartsuit) = \heartsuit^2 + 3\heartsuit - 1$$

$$f(3) = 2(3)^2 + (3) - 1$$

**Ex.** Using the examples above, convert the following equations into function notation.

a.  $y = 5x + 7$

b.  $g = 8h - 2$

c.  $b = -4d$

$$f(x) = 5x + 7$$

$$f(h) = 8h - 2$$

$$f(d) = -4d$$

### Evaluating Functions

When you want to know the output of a function, you can use your input values by substituting them into your function for the independent variable.

#### Evaluating Functions

$$f(x) = x + 1$$

$$f(2) = 2 + 1$$

**Ex.** Evaluate  $f(x) = 3x$  when  $x = 2$  and  $x = -8$

$$f(2) = 3 \cdot 2 = 6$$

$$f(-8) = 3 \cdot -8 = -24$$

**Ex.** Evaluate  $g(x) = \frac{1}{2}x - 3$  when  $x = -4$  and  $x = 8$

$$g(-4) = \left(\frac{1}{2}\right) \cdot (-4) - 3$$

$$= -2 - 3$$

$$= -5$$

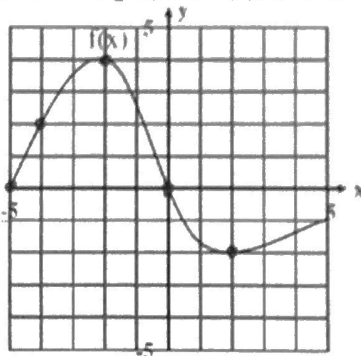
$$g(8) = \left(\frac{1}{2}\right) \cdot (8) - 3$$

$$= 4 - 3$$

$$= 1$$

### Evaluating a Function from a Graph

Given this graph of  $f(x)$ , evaluate the following:



a.  $f(-4) = +2$

b.  $f(0) = 0$

c.  $f(-5) = 0$

d.  $f(2) = -2$

e.  $f(0) = 0$

f.  $f(-2) = 4$

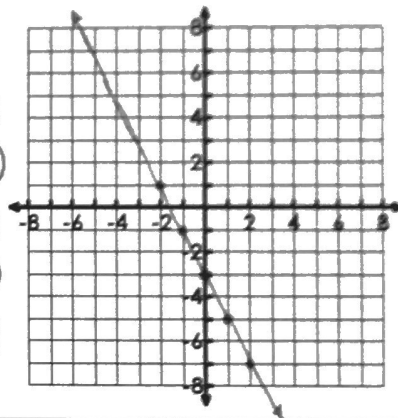
### Input and Output Tables and Graphing Functions

You can also evaluate functions to create input and output tables that can be used to graph the function

Ex. Using the values of -2, -1, 0, 1, and 2, complete the input/output table and graph.

x		y
Input	$f(x) = -2x - 3$	Output
-2	$f(-2) = (-2)(-2) - 3$ $= 4 - 3$	1
-1	$f(-1) = (-2)(-1) - 3$ $= 2 - 3$	-1
0	$f(0) = (-2)(0) - 3$ $= 0 - 3$	-3
1	$f(1) = (-2)(1) - 3$ $= -2 - 3$	-5
2	$f(2) = (-2)(2) - 3$ $= -4 - 3$	-7

- $(-2, 1)$
- $(-1, -1)$
- $(0, -3)$
- $(1, -5)$
- $(2, -7)$



### Testing if a Function is a Function (Vertical Line Test)

Another way to tell if a relation is a function is the **Vertical Line Test**. The Vertical Line Test is used with graphs of relations. To use the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of the relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

Ex. Use the Vertical Line Test to determine if the graphs of the relations are functions.

