Solve Quadratics by Factoring (All Methods)

Review: What would be the zeros in each equation below?

a. $(x-7)(x+5)=0$

$x-7=0$ $\quad x+5=0$

$+7 \quad +7 \quad -5 \quad -5$

$x=7 \quad x=-5$

b. $(x+4)^2=0$

$(x+4)(x+4)=0$

$x+4=0 \quad x+4=0$

$-4 \quad -4$

$x=-4 \quad x=-4$

c. $y=x(x-9)$

$0=x(x-9)$

$x=0 \quad x-9=0$

$x=9$

Factoring by GCF

When equations are in the form:

$Ax^2+Bx=0$

1. Rewrite the equation so it is set equal to 0.
2. Factor using the GCF.
3. Using the Zero Product Property, set each factor equal to 0.
4. Solve for the variable (isolate).

(Note: One zero (x-int) will always equal 0).

Factoring by Difference of Two Squares

When equations are in the form:

$Ax^2-B=0$

1. Rewrite the equation so it is set equal to 0.
2. Take the square root of term $\sqrt{Ax^2}$ and $\sqrt{B}$
3. Using the Zero Product Property, set each factor equal to 0.
4. Solve for the variable (isolate).

Factor and solve the following binomials:

a. $6x=2x^2$ Method: GCF

$-6x \quad -6x$

$0=2x^2-6x$

	x	-3
$2x$	$2x^2$	$-6x$

$\frac{2x}{2}=\frac{0}{2}$ $\quad x-3=0$ $\quad x=3$

Factored Form: $2x(x-3)$

Zeroes: $x=0,3$

b. $16x^2=4$ Method: 2 squares

$16x^2-4=0$

$\sqrt{16x^2} \quad \sqrt{4}$

$4x-2=0$

$+2 \quad +2$

$\frac{4x}{4}=\frac{2}{4}$

$x=\frac{2}{4}$ OR $\frac{1}{2}$

Factored Form: $(4x-2)(4x+2)$

Zeroes: $x=\frac{1}{2}$ or $-\frac{1}{2}$

c. $49x^2=25$ Method: 2 squares

$49x^2-25=0$

$\sqrt{49x^2} \quad \sqrt{25}$

$(7x-5)=0 \quad (7x+5)=0$

Factored Form: $(7x-5)(7x+5)$

Zeroes: $x=\frac{5}{7}, -\frac{5}{7}$

d. $3x^2=-18x$ Method: GCF

$3x^2+18x$

	x	$+6$
$3x$	$3x^2$	$+18x$

$3x=0 \quad x+6=0$

$x=0 \quad x=-6$

Factored Form: $3x(x+6)$

Zeroes: $x=0,-6$

Factoring trinomials when a = 1

When equations are in the form:

$$Ax^2 + Bx + C = 0$$

1. Rewrite the equation so it is set equal to 0.
2. Factor using x-method.
3. Using the Zero Product Property, set each factor equal to 0.
4. Solve for the variable (isolate).

Factoring trinomials when a ≠ 1

When equations are in the form:

$$Ax^2 + Bx + C = 0$$

1. Rewrite the equation so it is set equal to 0.
2. Factor out the GCF (if it is greater than 1).
3. Factor using x-method and if necessary, box method.
3. Using the Zero Product Property, set each factor equal to 0.
4. Solve for the variable (isolate).

Factor and solve the following trinomials:

a. $2x^2 + 3x = 2$ Method: $a \neq 1$

$2x^2 + 3x - 2 = 0$

① X-method: top −4, left 4, right −1, bottom 3

② Box method:

	x	2
2x	$2x^2$	4x
−1	−1x	−2

$2x - 1 = 0$
$+1 \quad +1$
$2x = 1$
$x = \frac{1}{2}$

$x + 2 = 0$
$x = -2$

Factored Form: $(2x-1)(x+2)$

Zeroes: $x = \frac{1}{2}, -2$

b. $x^2 + 3x = -2$ Method: $a = 1$

$x^2 + 3x + 2 = 0$

X-method: top 2, left 2, right 1, bottom 3

$(x+2) = 0$
$-2 \quad -2$
$x = -2$

$(x+1) = 0$
$-1 \quad -1$
$x = -1$

Factored Form: $(x+1)(x+2)$

Zeroes: $x = -1, -2$

c. $y = x^2 + 9x + 20$ Method: $a = 1$

$0 = x^2 + 9x + 20$

X-method: top 20, left 5, right 4, bottom 9

$(x+5) = 0$
$-5 \quad -5$
$x = -5$

$(x+4) = 0$
$-4 \quad -4$
$x = -4$

Factored Form: $(x+4)(x+5)$

Zeroes: $x = -4, -5$

d. $6x^2 + 3 = 11x$ Method: $a \neq 1$

$6x^2 - 11x + 3 = 0$

① X-method: top 18, left −9, right −2, bottom −11

② Box method:

	2x	−3
3x	$6x^2$	−9x
−1	−2x	+3

$3x - 1 = 0$
$+1 \quad +1$
$3x = 1$
$x = \frac{1}{3}$

$2x - 3 = 0$
$+3 \quad +3$
$2x = 3$
$x = \frac{3}{2}$

Factored Form: $(3x-1)(2x-3)$

Zeroes: $x = \frac{3}{2}, \frac{1}{3}$