Solving Systems Using Elimination

Another method for solving systems of equations when one of the variables is not isolated by a variable is to use **elimination.** Elimination involves adding or multiplying one or both equations until one of the variables can be eliminated by adding the two equations together. Elimination is also called linear combinations.

Take a look at the following systems of equations. Add the equations together and try to solve the system– what do you notice?

a. $3x + 2y = 7$
$+ \quad -3x + 4y = 5$
$6y = 12$
$y = 2$

$3x + 2(2) = 7 \quad (1, 2)$
$3x + 4 = 7$
$-4 \quad -4$
$3x = 3$
$x = 1$

b. $2x - 3y = 4$
$+ \quad -4x + 5y = -8$
$-2x + 2y = -4$

Steps for Solving Systems by Elimination

Step 1: Arrange the equations with like terms in columns.
Step 2: Analyze the coefficients of x or y. Multiply one or both equations by an appropriate number to obtain new coefficients that are opposites
Step 3: Add the equations and solve for the remaining variable.
Step 4: Substitute the value into either equation and solve.
Step 5: Check the solution by substituting the point back into both equation.

Elimination by Adding Systems Together

Ex 1. $-2x + y = -7$
$+ \quad 2x - 2y = 8$
$-y = 1$
$y = -1$

$-2x + (-1) = -7$
$-2x - 1 = -7$
$+1 \quad +1$
$-2x = -6$
$x = 3$

Solution: $(x, y) \rightarrow (3, -1)$

Ex 2. $4x - 2y = 2$
$+ \quad 3x + 2y = 12$
$7x + 0 = 14$
$7x = 14$
$x = 2$

$4(2) - 2y = 2$
$8 - 2y = 2$
$-8 \quad -8$
$-2y = -6$
$y = 3$

Solution: $(x, y) \rightarrow (2, 3)$

Elimination by Rearranging and Adding the Systems Together

Ex 3. $8x = -16 - y$
$3x - y = 5$
$+ \quad 8x + y = -16$
$11x + 0 = -11$
$11x = -11$
$x = -1$

rearrange: $8x = -16 - y$
$+y \quad +y$
$8x + y = -16$

$8(-1) + y = -16$
$-8 + y = -16$
$+8 \quad +8$
$y = -8$

Solution: $(x, y) \rightarrow (-1, -8)$

Ex 4. $2x + y = 8$
$-y = 3 + 2x$

rearrange: $-y = 3 + 2x$
$-2x \quad -2x$
$-2x - y = 3$

$2x + y = 8$
$-2x - y = 3$
$0 + 0 = 11$
$0 \neq 11$ FALSE!

Solution: No solutions

Elimination by Multiplying One Equation and Then Adding the Equations Together

Ex 5. $2(x + 12y = -15) \rightarrow 2x + 24y = -30$

$-2x - 6y = -6$

$+\ 2x + 24y = -30$

$0 + 18y = -36$

$\frac{18y}{18} = \frac{-36}{18}$

$y = -2$

$-2x - 6(-2) = -6$

$-2x + 12 = -6$

$-12 \quad -12$

$\frac{-2x}{-2} = \frac{-18}{-2}$

$x = 9$

Solution: $(x, y) \rightarrow (9, -2)$

Ex 6. $6x + 8y = 12$

$-3(2x - 5y = -19) \rightarrow -6x + 15y = 57$

$+\ 6x + 8y = 12$

$0 + 23y = 69$

$\frac{23y}{23} = \frac{69}{23}$

$y = 3$

$6x + 8(3) = 12$

$6x + 24 = 12$

$-24 \quad -24$

$\frac{6x}{6} = \frac{-12}{6}$

$x = -2$

Solution: $(x, y) \rightarrow (-2, 3)$

Ex 7. $-2(5x + y = 9) \rightarrow -10x - 2y = -18$

$10x - 7y = -18$

$+\ 10x - 7y = -18$

$\frac{-9y}{-9} = \frac{-36}{-9}$

$y = 4$

$10x - 7(4) = -18$

$10x - 28 = -18$

$+28 \quad +28$

$\frac{10x}{10} = \frac{10}{10}$

$x = 1$

Solution: $(x, y) \rightarrow (1, 4)$

Ex 8. $-1(7x + 2y = 24) \rightarrow -7x - 2y = -24$

$8x + 2y = 30$

$+\ 8x + 2y = 30$

$x + 0 = 6$

$x = 6$

$8(6) + 2y = 30$

$48 + 2y = 30$

$-48 \quad -48$

$\frac{2y}{2} = \frac{-18}{2}$

$y = -9$

Solution: $(x, y) \rightarrow (6, -9)$

Elimination by Multiplying Both Equations and Then Adding the Equations Together

Ex 9. $8(5x - 4y = -1) \rightarrow 40x - 32y = -8$

$-5(8x + 7y = -15) \rightarrow -40x - 35y = 75$

$+$

$\frac{-67y}{-67} = \frac{67}{-67}$

$y = -1$

$40x - 32(-1) = -8$

$40x + 32 = -8$

$-32 \quad -32$

$\frac{40x}{40} = \frac{-40}{40}$

$x = -1$

Solution: $(x, y) \rightarrow (-1, -1)$

Ex 10. $5(-6x + 12y = -6) \rightarrow 30x - 60y = 30$

$6(-5x + 10y = -5) \rightarrow -30x + 60y = -30$

$+$

$0 + 0 = 0$

$0 = 0$ ✓

TRUE!

Solution: Infinite Solutions

Ex 11. $2(-9x + 5y = 26) \rightarrow -18x + 10y = 52$

$9(2x + 2y = 16) \rightarrow +\ 18x + 18y = 144$

$\frac{28y}{28} = \frac{196}{28}$

$y = 7$

$2x + 2(7) = 16$

$2x + 14 = 16$

$-14 \quad -14$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

Solution: $(x, y) \rightarrow (1, 7)$

Ex 12. $3(2x + 2y = 10) \rightarrow 6x + 6y = 30$

$-2(3x + 5y = 13) \rightarrow -6x - 10y = -26$

$+$

$\frac{-4y}{-4} = \frac{4}{-4}$

$y = -1$

$3x + 5(-1) = 13$

$3x - 5 = 13$

$+5 \quad +5$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

Solution: $(x, y) \rightarrow (6, -1)$