Unit 2 Study Guide

Name: KEY

Block:

Proportional Reasoning & Dimensional Analysis

Learning Target #1: Proportional Reasoning

1. I can write a ratio in several ways.

Ratio: A comparison of two quantities using division
- *Order Matters when you write a ratio.
- *There are 3 ways to write a ratio (1:4, 1 to 4, ¼)
- *Always simplify your ratio

Your Turn

In Mrs. Dombrowski's class, there are 5 students who own an ipad and 15 students who own an iphone.

A. What is the ratio of iphones to ipads? 15 : 5, 15 to 5, $\frac{15}{5}$

B. What is the ratio of iphones to total students? 15 : 20, 15 to 20, $\frac{15}{20} = \frac{3}{4}$

For the following ratio, create a second part to whole and a part to part.

3 out of 10 prefer math over science class. (Part, Whole)

Part to Whole: 7 out of 10 prefer science over math.

Part to Part: 3 to 7 prefer math over science.

2. I can determine equivalent ratios by Scaling up or down.

Scaling up: multiplying numerator and denominator by the same factor
Scaling down: dividing numerator and denominator by the same factor.

$\frac{\text{problems correct}}{\text{minutes}} \rightarrow \frac{4}{5} = \frac{16}{20}$ (×4)

Find the missing values:

$\frac{2 \text{ blueberry muffins}}{5 \text{ total muffins}} = \frac{50 \text{ blueberry muffins}}{? \text{ total muffins}}$ (×25)

? total muffins = 5×25 = 125

$\frac{20 \text{ hours of work}}{\$240} = \frac{1 \text{ hour of work}}{?}$ (÷20) → $12

3. I can determine equivalent ratios using a table.

Ratio tables are helpful when solving word problems or if you are given a table with missing values. Realize that each column in the table represents a ratio and they are all equivalent (hence, you can also use a proportion to find the missing numbers).

Yellow paint (oz)	2	4	8	16
Blue paint (oz)	4	8	16	32

(×2, ×4)

A. Every 2 boxes of fruit snacks will serve 11 students. How many boxes can serve 22 and 33 students?

Boxes	2	4	6
Students	11	22	33

(×2, ×3)

4. I can determine equivalent ratios using proportions.

When solving proportions, you can scale up or down OR cross multiply.

You should LABEL everything in a proportion.

Sometimes, it is best to create your part to part ratio plus the two part to whole ratios before solving to help ensure you solve what is asked of you (See I Can Statement #1).

A. In a grade level, the number of boys to number of girls is 11:5. If there are 30 girls, how many **students** are there total?

$\frac{11 \text{ boys}}{5 \text{ girls}} = \frac{? \text{ boys}}{30 \text{ girls}} = 66 \text{ boys}$ (×6)

66 boys + 30 girls

96 total

B. For every 4 seniors, there are 9 freshmen. If there are 728 students total, how many of them are **seniors**?

$\frac{4 \text{ seniors}}{9 \text{ freshmen}}$

$4 + 9 = 13$ total

$\frac{4 \text{ seniors}}{13 \text{ total}} = \frac{224 \text{ seniors}}{728 \text{ total}}$ (×56)

5. I can determine a unit rate.

A unit rate is the rate for one unit of a given quantity which means they have a denominator of one.

Example: Sarah reads 88 pages in 4 hours. How many does she read in an hour?

$\frac{88 \text{ pages}}{4 \text{ hours}} = \frac{22 \text{ pages}}{1 \text{ hour}}$

Unit rates are also useful for determining better buys (which is cheaper per unit?).

Example: Is a 12 oz bag of chocolate chips for $4 a better deal than an 18 oz bag of chocolate chips for $4.89?

$\frac{\$4.00}{12\text{-oz}}$ ⇨ 12 ÷ 4 = $0.33 per ounce

$\frac{\$5.50}{18\text{-oz}}$ ⇨ 5.50 ÷18 = $0.31 per ounce —BETTER DEAL!

A. Austin travels 455 miles in 9 hours. How far did he go in one hour?

$\frac{455 \text{ miles}}{9 \text{ hours}} = \frac{50.6 \text{ miles}}{1 \text{ hour}}$ (÷9)

B. Which is the better deal? (unit rate)

12 oz bottle of Diet Coke for $1.09.

20 oz bottle of Diet Coke for $1.99

$\frac{\$1.09}{12 \text{ oz}} = \frac{\$0.91}{1 \text{ oz}}$ (÷12)

$\frac{\$1.99}{20 \text{ oz}} = \frac{\$0.10}{1 \text{ oz}}$ (÷20)

$\frac{\$0.91}{12 \text{ oz}} < \frac{\$0.10}{20 \text{ oz}}$

6. I can determine a Unit Rate from a Graph.

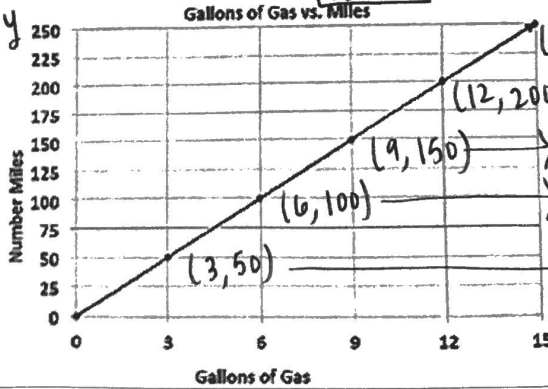
Each point on the graph represents a ratio that is equivalent to another ratio (point) on the graph.

The unit rate on a graph is the slope of the line.

When calculating the unit rate, it is the y-value over the x-value (rise over run).

Unit rates should always be labeled.

A. Calculate the unit rate: 16.67



unit rate = $\frac{y}{x}$

$\frac{y}{x} = \frac{150}{9} = 16.67$

$\frac{y}{x} = \frac{100}{6} = 16.67$

$\frac{y}{x} = \frac{50}{3} = 16.67$

Learning Target #2: PERCENTS

7. I can convert between Fractions, Decimals, & percents.

Fractions:
- To decimals: divide numerator & denominator
- To percents: turn to a decimal and move decimal two place values to the right

Decimals:
- To percents: move decimal two place values to the right
- To fractions: place over 10, 100, 1000 (depends on your place value)

Percents
- To decimals: move decimal two place values to the left
- To fractions: place over 100 and simplify(if necessary)

A. Show two other forms of the following numbers (percent, fractions, or decimal):

	Percent	Fraction	Decimal
1. 0.71	71%	71/100	—
2. 0.012	1.2%	$\frac{1.2}{100}$	—
3. 9%	—	$\frac{9}{100}$	.09
4. 245%	—	$\frac{245}{100}$	2.45
5. $\frac{2}{5}$	40%	—	.40 or .4
6. $\frac{5}{8}$	62.5%	—	.625

8. I can calculate the percent, Part, or Whole of a Number.

$$\frac{\text{Part}}{\text{whole}} = \frac{\%}{100}$$

The part is normally identified by the word "**is**".

The whole is normally identified by the word "**of**".

The whole indicates the total or original.

A. Calculate the part, whole, or percent of the following statement:

1. 8% of 40 is what number?

$.08 \times 40 = \boxed{3.2}$

2. 90 is 75% of what number? (Part / %)

$\frac{90}{x} \times \frac{75}{100}$

$100 \times 90 = 75x$

$\frac{9000}{75} = \frac{75x}{75}$

$\boxed{120 = x}$

check: $120 \times .75 = 90$ ✓

3. 3 is what percent (part) of 60 (whole)?

$\frac{3}{60} = .05 = \boxed{5\%}$

4. 40% of what number is 26? (part)

$\frac{26}{x} \times \frac{40}{100}$

$26 \times 100 = 40x$

$\frac{2600}{40} = \frac{40x}{40}$

$\boxed{x = 65}$

check: $65 \times .4 = 26$ ✓

9. I can apply percents to real world problems.

Tax and Tip are applications that are added to our total bill.
- I can treat the tax and tip like a "part" and add the tax to my bill.
- I can also treat them as an additional percent (100% + 7% tax rate = 107% of the bill), which calculates the final bill.

A. A baseball pitcher won 80% of the games he pitched. If he pitched 35 games, how many did he win?

80% of 35

$.8 \times 35 = \boxed{28 \text{ games}}$ OR $\frac{x}{35} \times \frac{80}{100}$

$100x = 2800$

$\boxed{x = 28 \text{ games}}$

B. The Gap is selling shirts for 30% off the original price. The shirts are on sale for $22.40. What was the original price of the shirt?

sale price (30% off original, so 70% of original)

$\frac{\$22.40}{.7} = \boxed{\$32.00}$

check: $\$32 \times .3 = \$9.6$

$\$32 - \$9.6 = \$22.40$ ✓

Discounts are applications that are subtracted from original price.

- If I calculate using the **percent off**, the number I get out is the discount that I should subtract from the original price.
- If I calculate using the **percent paying**, the number I get out is the price I would pay for the item after the discount. (100% - 25% off = 75% paying for the item).

I can use the percent proportion to solve real world problems.

C. A bike is on sale for $162.50 If the original price of the bike is $250. what is the percent discount?

① $250 − $162.50 = $87.50
original price − sale price = discount price

percent discount = $\frac{\text{discount price}}{\text{original price}} = \frac{\$87.50}{\$250}$ ② = .35 = 35%

D. A pair of sunglasses costs $26.00 during a summer sale. If there is a 6% sales tax and Daily pays for the glasses with $30. how much change should she expect?

① $26.00 × .06 = $1.56
original price, tax

② Total paid = $26 + $1.56 = $27.56

③ change = $30 − $27.56 = $2.44

F. You go out to eat with your family and have a delicious meal at Mellow Mushroom. Your total bill, including a 20% tip comes to $51.47. What was the tip?

① $51.47 × .20 = $10.29
Total paid, % tip, $ tip

G. The shirt you have been wanting for the ~~past 3 months~~ goes on sale for 25%. When you arrive to the store, you find out there is an additional 10% off the sale price of the shirt. If the shirt is originally $45, what is the final sale price of the shirt?

① $45 × .25 = $11.25
original price, % discount, sale price

② $11.25 × .10 = $1.13
sale price, % discount, price discount

③ final sale price = $11.25 − $1.13 = $10.12

Learning Target #3: Dimensional Analysis

10. I can convert within the metric system.

A. Convert the following:

1. 12.54 km = 12,54000 (1,254,000) cm Kilo → centi KHDBDCM 5 right

2. 457 mL = .00457 hL milli → hecto KHDBDCM 5 left

.00457

KHDBDCM
2 right

k	h	d	U	d	c	m
kilo	hecto	deka	UNIT GRAM METER LITER	deci	centi	milli

When moving the decimal to the left, you are dividing by a power of 10.

When moving the decimal to the right, you are multiplying by a power of 10.

When comparing two quantities, make sure they are in the same unit before comparing (you might have to convert one of them to the other unit first).

3. 0.55 dag = 55 dg deka → deci

B. Compare the following

4. 7.225 cm = 72.25 m

Centi → meter (base)

KHDBDCM
2 left

7,225

72.25m = 72.25m

5. 34 g = 0.34 hg

gram (base) → hecto

KHDBDCM
2 left

.34

.34hg = .34hg

11. I can convert units of measure (1 & 2 step).

Conversion Factor: $\frac{\text{what you want}}{\text{what you have}}$

If you are going from Metric to Customary or vice versa, the conversion factor will be given to you.

A. Convert 5 miles to feet. (1 mile = 5,280 feet)

$5 \text{ miles} \times \frac{5{,}280 \text{ feet}}{1 \text{ mile}} = 26{,}400 \text{ Feet}$

B. Convert 4 years into days. (1 year = 365 days)

$4 \text{ years} \times \frac{365 \text{ days}}{1 \text{ year}} = 1460 \text{ days}$

C. How many miles will a person run during a 10 kilometer race? (1.6 kilometers in one mile) Kilometer → mile

$10 \text{ Kilometers} \times \frac{1 \text{ mile}}{1.6 \text{ Kilometers}} = \frac{10 \text{ miles}}{1.6}$

$= 6.25 \text{ miles}$

12. I can convert units of measure (Multi-Step & Word Problems).

Make sure you write every single conversion factor!

Think about where you are starting and where you want to go. Create a plan that includes the necessary conversion factors.

A. Convert 12 pints to gallons. pints → quarts → gallons

$12 \text{ pints} \times \frac{1 \text{ quart}}{2 \text{ pints}} \times \frac{1 \text{ gallon}}{4 \text{ quarts}} = \frac{12 \text{ gallons}}{8}$

$= \frac{3}{2}$ gallons OR 1.5 gallons

B. Sarah ran a 10 meter race. How many feet is that? (1 in = 2.54 cm) meters → centimeters → inches → feet

(base)
meters → centimeters
KHDBDCM
2 right

10 meters → 1,000 centimeters

$1{,}000 \text{ centimeters} \times \frac{1 \text{ inch}}{2.54 \text{ cm}} \times \frac{1 \text{ foot}}{12 \text{ inches}}$

$\frac{1{,}000 \text{ feet}}{2.54 \times 12} = \frac{1{,}000 \text{ ft}}{30.48} = 32.8 \text{ feet}$

Example: A bucket has 4.65 L of water. How many gallons of water is that (1.06 qt = 1 L).

Given: 4.65 L **Needed:** gallons

Plan: L ⟶ qt ⟶ gallon

Equalities: 1.06 qt = 1 L; 1 gal = 4 qt

Set Up Problem:

$$4.65\,\text{L} \times \frac{1.06\,\text{qt}}{1\,\text{L}} \times \frac{1\,\text{gal}}{4\,\text{qt}} = 1.23\,\text{gal}$$

C. A bowl of cereal weighs 60 oz. How heavy is it in kg? (1 oz = 28.3 g)

oz → grams → Kilograms (base)

$$60\,\text{oz} \times \frac{28.3\,\text{grams}}{1\,\text{oz}} = 1{,}698\,\text{grams} \rightarrow \text{Kilograms}$$

KHDBDCM — 3 left

1.698 = **1.698 Kilograms**

13. I can perform a rate conversion.

Sometimes it is helpful to convert either the numerator or denominator first and then convert the other. If you do too much at once, your problem gets messy.

Example: Convert 66 feet per second to miles per hour.

$$\frac{66\,\text{feet}}{1\,\text{sec}} \cdot \frac{60\,\text{sec}}{1\,\text{min}} \cdot \frac{60\,\text{min}}{1\,\text{hour}} \cdot \frac{1\,\text{mile}}{5280\,\text{feet}} = 45\,\text{miles/hour}$$

A. Convert 65 mph to feet per minute. (miles per hour)

miles → feet

hour → minute

$$\frac{65\,\text{miles}}{1\,\text{hour}} \times \frac{5{,}280\,\text{feet}}{1\,\text{mile}} \times \frac{1\,\text{hour}}{60\,\text{minutes}} = \frac{65 \times 5{,}280\,\text{feet}}{60\,\text{minutes}} = \frac{5{,}720\,\text{feet}}{1\,\text{minute}}$$

B. Convert 32 feet/second to meters/min (1 inch = 2.54 cm).

feet → inches → centimeters → meters

seconds → minutes

④ KHDBDCM — 2 left

① $\frac{32\,\text{feet}}{1\,\text{second}} \times \frac{12\,\text{inches}}{1\,\text{foot}} \times \frac{2.54\,\text{cm}}{1\,\text{inch}} \times \frac{60\,\text{seconds}}{1\,\text{minute}}$ → $\frac{585.216\,\text{meters}}{1\,\text{minute}}$

② $= \frac{32 \times 12 \times 2.54 \times 60\,\text{cm}}{1\,\text{minute}}$ ③ $= \frac{58{,}521.6\,\text{cm}}{1\,\text{minute}}$ → meters (base)

C. The average American student is in class 330 minutes/day. How many hours per school week is this (use 1 school week = 5 days)?

minutes → hours

days → school week

$$\frac{330\,\text{minutes}}{1\,\text{day}} \times \frac{1\,\text{hours}}{60\,\text{minutes}} \times \frac{5\,\text{days}}{1\,\text{week}}$$

$$= \frac{330 \times 5\,\text{hours}}{60\,\text{weeks}} = \frac{1650\,\text{hours}}{60\,\text{weeks}} = \frac{27.5\,\text{hours}}{1\,\text{week}}$$